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THE CALCULATION OF LATERAL STABILITY WITH FREE CONTROLS

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SUMMARY

Every report on disturbed lateral motion published heretofore in German as well as in foreign technical literature, stresses the mathematical treatment of the whole asymmetric motion processes, inclusive of the control effects. The practical requirements for the airplane designer are in most instances only ~~promiscuously~~ touched upon without any further details.

The increased safety requirements, particularly for commercial airplanes, made it seem necessary to give the airplane manufacturer in simple language an explanation of the theory and of the ensuing structural requirements.

The applications to modern airplane designs necessitated a corresponding modification of several of the old theorems. This applies particularly to the introduction of the constant "fuselage proportion" in the directional stability $(c'_{m_{s_0}})$, to the constant "wing proportion" in damping in yaw $(c'_{d_{s_0}})$, and to the constant roll stability quota for the complete wing without dihedral $(c'_{m_{q_0}})$. On the other hand, a detailed discussion of the effect of forward vertical tail surfaces could be omitted in this connection.

With the revised equilibrium formulas, the most important phase of lateral motion, i.e., the lateral stability, could then be treated. The formal derivation of the sta-

*"Die Seitenstabilität des ungesteuerten Normalfluges und ihre technischen Vorbedingungen." Z.F.M., April 14, 1932, pp. 193-199; and April 28, 1932, pp. 224-232.

bility conditions offers nothing fundamentally new, but contrary to Fuchs and Hopf (reference 6), we retained the effect of the path inclination up to the formation of the stability equations in the development.

The complete discussion of the general motion following a disturbance could be omitted as it is not of great significance so far as the airplane manufacturer is concerned. As to the factory test pilot, his primary interest lies with the most important sign of instability: the spiral curve with increasing curvature. Other signs of insufficient dynamic stability hitherto less noticed because less frequent and omitted by Fuchs and Hopf, are the unstable oscillations, simultaneous rotary oscillations of the airplane in yaw and roll, and center-of-gravity oscillations about the direction of the main path. This form of motion is a result of insufficient directional stability and can become the more evident as the roll stability is higher. The separate determination of directional and roll stability is - aside from the difficulty of conclusive proof in flight - no satisfactory characteristic for lateral stability. On the contrary, it must be complemented by observation of the motion process following a lateral disturbance with free or neutrally fixed controls. The lateral stability may be considered as proved when, following a disturbance, the airplane tends to reestablish its initial condition of equilibrium aperiodically with decreasing path curvature.

The discussion of the structural methods for obtaining lateral stability discloses the remarkable influence of the constant fuselage and wing proportions to the yawing moments. For the effectiveness of modifications in vertical tail surfaces and tail length these quotas - little observed heretofore, in this connection - are decisive. This also applies to the amount of dihedral of the wing with regard to the roll stability of the complete wing already existing without angle of dihedral.

Because of the constant moment quotas, markedly smaller dihedral angles are more nearly always sufficient than Reissner's well-known old approximation formula (reference 2) led one to anticipate. The data for its amount can be obtained from the evaluated 6-component measurements in the wind tunnel.

The structural means which foster lateral stability

are: positive roll stability (dihedral or sweepback) and smallest possible rolling moments in yaw (appropriate plan form or twist of wing); then, strongest possible damping in yaw (long tail, large fin and rudder) and moderate relative directional stability (attainable even with good absolute directional stability through inherently unstable fuselage with large fin and rudder).

The advantage of lateral stability for control of the airplane cannot be judged from the point of view that the laterally stable airplane is perhaps able to maintain of its own accord a once-existent course direction, just as little as a fore-and-aft stable airplane is expected to keep flight altitude of itself. Course direction and flight altitude are purely navigation concepts which cannot have any direct bearing on the airplane which follows the aerodynamic and mass-mechanical laws. Besides, it is immaterial whether the steady equilibrium condition is a straight or a flat, curved path; in pitching, for instance, the equilibrium condition may equally be with level or inclined flight path. Asymmetries in equilibrium condition can be modified with the ailerons and the vertical fin as readily as nose or tail heaviness can be corrected by deflecting the stabilizer. Then, too, the effects of engine torque and propeller slipstream or the stoppage of a wing (outboard) engine of multi-engine airplanes occur as symmetry interferences in the lateral motion, and as load distribution changes in another corresponding manner in the longitudinal motion.

The value of the lateral as of the longitudinal stability lies in the fact that flight attitude - once obtained with the customary auxiliary means (fin adjustment, control balancing, etc.) - is automatically maintained without action of the pilot and automatically reestablished after every disturbance. The type of initial equilibrium condition is of no moment; all flight attitudes occurring within range of moderate disturbances of symmetry, symmetrical straight flight included, must be uninterruptedly stable. In this case, maintaining the course and altitude is also markedly easier for the pilot, although never to the extent of making special instruments altogether superfluous.

It is not the function of automatic control devices to substitute for insufficient inherent stability of the airplane but rather to aid the existent inherent stability

by control movements. Ready and quiet operation is contingent upon the premise that the pertinent control settings tend to return the airplane ordinarily toward steady equilibrium conditions against the direction of the disturbing motion. But this condition is met only with inherent airplane stability and is its characteristic.

Controllability and stability are closely related; both are usually interwoven in the theoretical treatment, especially in the subject of lateral motion. But from the point of view of the practical flyer, it seems more useful to elucidate the stability relations before attacking the problem of controllability. The investigations available thus far justify this order of sequence insofar as they have proved the favorable effect of the stability on the controllability. For this reason it may not be amiss to include hereinafter (as references) a list of the reports on Lateral Stability. (See page 43.)

I. INTRODUCTION

The consistent growth of German commercial aviation supplied the necessary impetus for greater attention to flight performance from the point of view of safety. Beginning with the range of longitudinal motion, it was found that ample longitudinal stability is today a generally admitted requisite safety measure. But a definitely satisfactory lateral stability is still in its stages of development, although its theoretical aspect has been known for nearly twenty years. Admittedly, the combination of stability in yaw and roll obtaining with lateral stability presents an obstacle for the airplane designer which does not exist with longitudinal stability in this form.

Little importance attaches to lateral stability so long as the pilot can visibly perceive any change in airplane attitude with respect to the natural horizon. The growth of a disturbing motion accompanying the usually small lateral instability is so slow that minor control movements, sometimes almost subconsciously, suffice to reestablish the steady equilibrium attitude. But in "blind flying" the pilot does not become conscious of changes in bank and rate of yaw except by reading the turn indicator. The previously tactually executed corrections then become

intentional maneuvers. In the laterally unstable airplane the turn indicator becomes an instrument for "balancing" the disturbances of an unstable flight attitude, whereas its failure removes any reliable assurance for the avoidance and timely correction of dangerous attitudes. It is here that the lateral stability can contribute to a perceptible relief of the pilot by automatically removing small disturbances. The importance of the turn indicator and the banking level for the lateral control is here limited - similarly to that of the dynamic pressure recorder and the pitching indicator - to the control of the longitudinally stable airplane. In both cases the instruments reveal primarily the kind of momentarily existing condition of equilibrium; minor deviations from the original attitude may disappear again without action on the part of the pilot. The aim of the development must be to enable an airplane to maintain its attitude of equilibrium more evenly by means of its inherent stability than afforded by control movements.

The reports published heretofore on lateral stability stress the mathematical aspect and the whole course of motion, as a result of which the airplane manufacturer is forced to critically analyze almost the entire material before he can hope to obtain any data which may be of use to him. This is the reason the available data have not as yet become general knowledge in professional circles. The present report intends to separate the technically important facts from the mathematical theory and to make them more easily understood in a form familiar to the aeronautical engineer.

Scope.- The stability in yaw relative to minor disturbances of equilibrium attitude can usually be analyzed as distinct from the motion in roll. This applies in particular to level flight and flat turn at medium angles of attack and small angles of bank and sideslip. The motions in turns with considerable bank may not be summarily included in such simple fashion, and they are omitted in this report.

The slope of the flight path to the horizontal plane may be included in the simplified analysis of yawing motion without rendering it substantially more difficult. This obviates the limitation to level flight which obtains in most German reports.

Whether flying with hands off or on the controls, following an outside disturbance the airplane executes only the free balancing motion prescribed by its inherent stability. With control displacements these balancing motions are superposed on the forced transition in the new equilibrium attitude, which corresponds to the new control settings. As a result the analysis of inherent stability need not include any detailed discussion of the related problems of controllability and may be limited to the free motions about a definite, steady attitude of equilibrium.

The principal axes of inertia of the airplane are used to denote the body axes of the coordinate system, to conform with the conventional German system of notation (reference 6). An exception is the positive sense of direction of the longitudinal axis which here is chosen as pointing rearward, as explained elsewhere in the report.

II. DISTURBANCES OF LATERAL MOTION

Such disturbances are followed by the change of three quantities, whose initial value is representative of the type of initial attitude: the rate of sideslip of the plane of symmetry of the airplane in direction of the pitching axis, the rate of rotation about the axis of yaw (rate of yaw), and the rate of rotation about the axis of roll (rate of roll). The most elementary case is perhaps the straight flat glide without sideslip with almost zero propeller thrust. In other normal initial attitudes the line of argument is the same for the disturbing motion, which is simply superposed on the initial motion.

The initial conditions for straight steady flight without banking and sideslipping is:

Rate of sideslip	= 0	for axis of pitching,
Rate of yaw	= 0	for axis of yaw,
Rate of roll	= 0	for axis of roll.

Assume the source of disturbance is a sudden rate of sideslip. The effect of the induced air loads on the airplane depends on the relation of the rate of sideslip in direction of axis of pitching to the forward speed in direction of the axis of roll. This ratio, which is usually small in a disturbance without control movement, represents

the angle of sideslip between the symmetrical plane of the airplane and the resultant flight path direction, which herewith occurs as real decisive variable in place of the rate of sideslip.

The angle of sideslip causes a cross-wind force transverse to the flight path as well as rolling and yawing moments. The thus promoted yawing motion of the airplane then engenders further rolling moments as a result of the difference in wing-tip speed, and this change in banking influences in turn the further course of sideslipping. Consequently, rate of sideslip, rate of yaw, and rate of roll are intimately related in disturbances and may not be analyzed separately. Nevertheless, the first assumes a leading role by virtue of its direct effect on the aerodynamic loads. Any lateral disturbance of any source whatsoever, ends in changed angle of sideslip which in turn arouses the stabilizing moments.

Yawing and rolling moments due to sideslip.- The generation of restoring moments following the disturbance of equilibrium about one of the airplane axes, independently of the acceleration attitudes in the other course of motion, is representative of its static stability.

For the normal (yaw) axis, cause and effect are immediately clear: An airplane is statically stable in yaw when the moments of the air loads, due to angle of sideslip as a result of yawing, tend to return the airplane in the direction of the air stream. The English call this quality "weathercock stability", while in Germany, it is known as "directional stability". This term is permissible when bearing in mind that it only pertains to the direction of the plane of symmetry of the airplane relative to the air stream but not with respect to its heading relative to the horizon.*

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*Having no physical justification, it is misleading to speak of restoring yawing moments as being indicative of directional stability. No airplane is stable in yaw per se. The use of a compass with automatic course control may effect a stabilization of the direction. This kind of course stabilization represents no airplane characteristic, but rather the transfer of a quality of the auxiliary instrument to the airplane control.

The amount and direction of the yawing moments are controlled by the design of wings, fuselage, and vertical tail surfaces. With asymmetrical air flow, the wing produces yawing moments whose magnitude depends on its plan form and the dihedral (reference 17). The component of the fuselage comprises all quotas of the structural components which by their presence may produce stabilizing or unstabilizing aerodynamic moments, such as engine mounts, wheels, floats, etc. The moments of the vertical control surfaces, in the sense of the ensuing stability analysis, originate as air-load effect on those guide surfaces which as aerodynamically independent cross-wind units, extend beyond the top (at times also below the bottom) edge of the side area of the fuselage. Airplanes with very high rear fuselage edge have according to this concept in most cases a very small tail area, but at the same time a more or less pronounced inherent stability of the fuselage in yaw.

In modern designs the applied moment of the air loads at the fuselage usually lies ahead of the c.g. of the airplane, so that in sideslipping the fuselage generally contributes an unstabilizing component, in absolute amount about equal to that of the stabilizing vertical tail surfaces of conventional size and arrangement. The amount of directional stability can for that reason become very sensitive against displacements of the mass center of gravity in longitudinal direction of the fuselage.

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As to the longitudinal axis, we cannot speak of direct static stability except with qualifications. After a rotation of the airplane about its longitudinal axis (roll), its plane of symmetry is generally no longer flight pathwise. The deviation is, at small angles, proportional to the angle of attack between longitudinal axis and flight path and the angle of bank, and for the moments at the airplane it is equivalent to sideslipping. In the here discussed normal-flight attitudes with very small angles of attack between longitudinal axis and flight path this asymmetry can be disregarded against the actual angles of sideslipping, but at greater angles this omission is apparently no longer justified.

Turning the plane of symmetry of the airplane out of the gravity direction, disturbs the equilibrium of the cross-wind forces and promotes sideslipping, through which the restoring rolling moments may be aroused. These moments are typical of static "stability in roll." Their

inception as with the directional stability is contingent upon the appearance of an angle of sideslip between the plane of symmetry of the airplane and the direction of the flight path.

The stabilizing rolling moments are predominantly supplied by normal forces on a wing of appropriate design (dihedral, sweepback) (reference 17). The effect of the cross-wind forces applied above or below the center of gravity is relatively subordinate. The angle of sweepback being fixed within narrow limits by the conditions of longitudinal stability and other design measures, the dihedral is the only design quantity which affords an effective influence of the stability in roll. Sideslipping on a wing with dihedral acts as an increase in angle of attack on the advancing half and as a decrease on the following half wing. The rolling moment due to this asymmetry depends upon the aerodynamic qualities of the wing. For different plan forms it may in first approximation be made proportional to the distance of the geometrical center of gravity of a half wing from the longitudinal axis (s_u).

This c.g. distance is, for several simple plan forms, as follows:

Constant chord: $\frac{s_u}{b/2} = \frac{1}{2} = 0.50$

Elliptic in chord: " $= \frac{4}{3\pi} \approx 0.42$

Parabolicin chord: " $= \frac{2}{5} = 0.40$

Triangular in chord: " $= \frac{1}{3} \approx 0.33$

In practice the value $\frac{s_u}{b/2} = 0.4$ represents approximately the usual lower limit.

The concurrent effect of an angle of sideslip on the yawing and rolling moments of an airplane is decisive for the entire yawing motion. The stability in yaw - even in unaccelerated path motion - must be insured not only through the existence of stability in roll and yaw but also through their correct mutual accord.

Wing moments due to yaw and roll.- In undisturbed level flight the air loads are distributed spanwise. Any asymmetry at the inside portion of the wing owing to fuselage and slipstream effect may be disregarded because their static moments about the c.g. of the airplane are small compared to the proportions of the outer wing parts. The static moment of a half wing depends on the lift quota per unit length of span, the lift density. This is contingent upon the locally effective circulation strength, that is, the locally effective angle of attack and the air stream velocity together with the corresponding wing chord. The "ideal" spanwise lift density is, as known, elliptic, although in practice the lift distribution ranges from rectangular to parabolic.

Yawing and rolling is followed by deflections of the lift density to unsymmetrical forms, that is, spanwise unsymmetrical change of air stream velocity and direction. The ensuing moments are of fundamental importance for the entire course of disturbed motion.

The moments due to yaw are primarily the result of changed air stream velocity, which increases at the advancing half and decreases at the following half of the wing. The yawing moment is an effect of the modified tangential load distribution and sets up a damping in yaw, whereas the rolling moment due to the changed normal load distribution has an unstabilizing effect.
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Both moments are proportional to the rate of yaw, the forward speed, the wing area, and the square of the span. They also manifest a marked dependence on the type of the original spanwise load distribution. Fuchs and Hopf (reference 6) considered only the most elementary limit case of rectangular air-load distribution. According to this assumption, the result in the majority of practical cases is an unduly large rolling moment (up to 30 percent) because of the disregarded lift decrease at the wing tips. The error for the yawing moment is usually less, as the proportion of the induced drag rather reveals a density rise in the outer portions of the span (reference 19). The magnitude of both moments depends on the geometrical radius of gyration (i) of the air load distribution figures. For comparison we append several simple lift distributions for a half wing:

Rectangle: $\left(\frac{i}{b/2}\right)^2 = \frac{1}{3} \approx 0.33$

Ellipse: " $= \frac{5}{16} \approx 0.31$

Semiellipse: " $= \frac{1}{4} \approx 0.25$

Parabola: " $= \frac{8}{35} \approx 0.23$

Rectangular wings with high aspect ratio show rectangular distribution at small angles of attack; the ellipse on each half-wing may approximately show a disturbed distribution in the center, the semiellipse corresponds to the "ideal" distribution, whereas parabolic distributions occur, for example, on extremely tapered or twisted wings. A more detailed investigation of the tangential load distribution is, except for tailless airplanes, mostly not worth while because in yawing the damping effect of the wing recedes, apart from that, relative to the damping in yaw on account of fin and rudder. The premise of uniform tangential load density (rectangle) should suffice in practically all cases for a satisfactory estimation.

A roll sets up wing moments as a result of the changed air-stream direction; the concurrent change in velocity is subordinate. The modified angle of attack for the individual wing parts follows from the vector sum of the peripheral speeds about the longitudinal axis of the airplane with the forward speed. The rolling moment effects a damping in roll; at the same time the unsymmetry of the tangential forces causes an additive yawing moment during the roll.

These two moments are proportional to the rate of roll, the forward speed, the wing area, and the square of the span, in addition to the plan form and the aerodynamic characteristics of the wing. Here also Fuchs and Hopf (reference 6) give only the most elementary limit case of spanwise uniform increase of air-load coefficients for the unit change of air-stream direction. The omission of the boundary effect, particularly for the rolling moment, again results in misleading figures (as much as 20 percent too high). Data for an approximate calculation of damping in roll with certain wing forms and elliptic lift distribution may be found in N.A.C.A. Technical Report No. 200 (reference 18).

and DVL report (reference 22). As to the yawing moment, the error should perhaps be less significant. The possibilities of the initial tangential load distribution and its change with the angle of air flow are so difficult to survey, that the estimation of a mean rise of tangential load for the whole span is almost imperative. Since this has a positive sign at small lift figures but generally a negative sign at medium and large lift figures, the yawing moment acts in normal flight attitudes mostly toward a forward rotation of the half wing receding in the motion of roll. In any case it is always relatively small compared to the other yawing moments due to yaw, so that a small percentage error in estimation should have no appreciable effect on the results.

With ailerons released the wing moments are dependent on the momentary angles of deflection which the ailerons assume during the motions. As the weight moments are to balance within the aileron control and the control friction is to be negligible, the sum of the aerodynamic moments at both ailerons must become zero. Apart from the elevator setting the aileron moments are dependent on: profile form, plan form, and dihedral of wing; plan form, balance, and differentiation of ailerons; angle of attack, sideslip, rate of yaw, and rate of roll at the pertinent moment of the whole flight motion.

All these quantities are mutually and in part very closely related, which renders linear or other elementary analytical formulas inapplicable. For this reason, the inclusion of free ailerons in the equilibrium equations for lateral motion (yaw) must be omitted.

The equilibrium equations of yaw. a) Cross-wind forces.- Supposing that for any reason of disturbance the airplane, flying level, assumes a dissymmetric motion, its initial amount being denoted by a bank μ with respect to the horizon and a sideslip τ relative to the air flow.

In a bank the plane of symmetry of the airplane is no longer gravity-wise, with the result that the acceleration on the c.g. of the airplane acts with a lateral component corresponding to $\sin \mu$. In an initially and subsequently assumed constant slope of the flight path relative to the horizon of amount φ_0 this lateral gravity component, perpendicular to the flight path, amounts to

$$Z_\mu = G \cos \varphi_0 \sin \mu$$

or for a gentle bank,

$$Z_\mu = G \cos \varphi_0 \mu \tag{5}$$

The angle of sideslip causes side loads on the airplane, the amount of which is contingent upon the sides of the fuselage and other similar parts, the wing setting, and the area of fin and rudder. However, these side loads are not of decisive importance for the total course of the ensuing motion; although they may, at the start of the disturbance, cause in airplanes with large fuselage sides, as favored in modern design practice, play a noticeable role for the force equilibrium. Without it, a "sideslip curve" with gentle bank is inconceivable.

In sideslipping with power on, there is an added component of the propeller thrust S, athwart the flight path amounting to

$$S \sin \tau \approx \left(c_w F \frac{\rho}{2} v^2 + G \sin \varphi_0\right)\tau$$

The total side load due to sideslipping at moderate sideslip angles τ is

$$Z_\tau + G \sin \varphi_0 \tau =$$

$$\left[\left(-\frac{\partial c_{q_0}}{\partial \tau} F_0 + \frac{\partial c_{n_s}}{\partial \alpha_s} F_s + c_w F\right)\frac{\rho}{2} v^2 + G \sin \varphi_0\right]\tau \tag{6}$$

Owing to the side loads Z_μ and Z_τ due to banking and sideslipping, there is a deviation from the hitherto existing flight-path direction. The airplane instead of flying level now, goes into a flat curve, whose radius with the time rate of change in route direction (rate of turn $\dot\psi$) and the horizontal component of the path velocity is $(v \cos \varphi_0)$. As a result, there is a lateral centrifugal force

$$Z_{\dot\psi} = \frac{G}{g} v \cos \varphi_0 \dot\psi \tag{7}$$

at the c.g. of the airplane.

During the motion of disturbance there are other side loads due to the angular velocities about the airplane axes (damping loads). The damping loads due to yaw are negligi-

ble relative to the centrifugal force. Those due to roll may, with pronounced wing dihedral, reveal mathematically tractable lateral components, although with the customary dihedral angles, they are of negligible order of magnitude.

Now the equilibrium equations of the cross-wind forces with due allowance as to direction are

$$\Sigma Z = Z_\psi - (Z_T + G \sin \varphi_0 \tau) - Z_\mu = 0 \tag{8}$$

wherein the individual loads are written conformably to (5), (6), and (7).

b) Yawing moments.- During an asymmetrical motion of the airplane the angle of sideslip causes yawing moments whose direction in a statically unstable airplane tends to an increase, in a statically stable airplane to a decrease of the angle of sideslip. These moments, principally due to fuselage and vertical tail surfaces, are typical of the directional stability of an airplane; they follow the equation

$$L_T = [c'_{m_{s_0}} F b + c'_{n_s} F_s l_s] \frac{\rho}{2} v^2 \tau \tag{1b}$$

The yaw causes damping moments proportionate to the rate of yaw ω_y. The predominant proportion is due to the vertical tail surfaces which during the rotation experience a change in air-speed direction amounting to $\frac{l_s \omega_y}{v}$. Added to this is the mathematically difficult tractable proportion of the fuselage sides rotating about the c.g. of the airplane, small for inherently stable fuselage with high rear edge, but perhaps worth considering at the tail end. The wing also contributes its quota, which with large span or tailless types may be effectively noticed. The fuselage quota can be simply added to the usually much greater wing quota; both together form a "damping factor":

$$c_{d_{s_0}} = \frac{1}{2}\left[c_t \left(\frac{l_t}{b/2}\right)^2 + \frac{(L_{\omega_y})_{\text{fuselage}}}{F\, b\, \frac{b \omega_y}{2 v} \frac{\rho}{2} v^2}\right].$$

The total damping in yaw is expressed as

$$L_{\omega_y} = [c_{d_{s_0}} \; F \; b^2 + c_n' \; F_s \; l_s^2] \; \frac{\rho}{2} \; v \; \omega_y \qquad (9b)$$

During roll the asymmetrical distribution of the spanwise tangential loads sets up a further yawing moment which, according to a previous section (page 10), makes this "yawing moment in roll" tractable to

$$L_{\omega_x} = c'_{(t)} \; F \; \frac{b^2}{12} \; \frac{\rho}{2} \; v \; \omega_x \qquad (4b)$$

The proportion of the vertical tail surfaces may be disregarded, first because it is negligible, and second, because of the uncertainty of its air flow as affected by the rotation of the wing.

The aerodynamic moments L_T, L_{ω_y}, L_{ω_x} must balance the mass moments due to yaw. The equilibrium equations of the yawing moments then read as

$$\underline{\Sigma \; L = \frac{G}{g} \; i_y^2 \; \dot{\omega}_y + L_{\omega_y} + L_T - L_{\omega_x} = 0} \qquad (10)$$

The aerodynamic moments are inserted according to (1b), (9b), and (4b).

c) <u>Rolling moments</u>.- In the presence of static stability the sideslip following a change in bank sets up a rolling moment which tends to nullify the produced bank. This moment is almost entirely caused by the wing at which with appropriate design the angle of sideslip produces asymmetrical changes in lift distribution. This effect of the lateral stability is expressed with

$$K_T = \left[c'_{m_{q_0}} + \frac{1}{2} \; \frac{c'_{a_\infty}}{1 + \dfrac{2c'_{a_\infty}}{\pi \Lambda}} \; \frac{s_u}{b/2} \; \nu \right] F \; b \; \frac{\rho}{2} \; v^2 \; \tau \qquad (2)$$

The yaw sets up a rolling moment as a result of the unequal speed of the two wing halves. This asymmetry tends to tip the airplane toward the inside of the yaw. According to a preceding section (page 10), it should also be possible to promote such an effect for the horizontal tail surfaces, which because of negative lift would largely

oppose the rolling moment of the wing. Moreover, a high vertical tail group could contribute a proportion in the same direction as the wings. But the proportions of this tail group are so small compared to the wing moment and besides, so uncertain, owing to the effect of wing and fuselage that their inclusion would not be justified. Thus the "rolling moment in yaw" may be accurately enough expressed with

$$K_{\omega_y} = \frac{1}{2}\, c_n\, F\, b^2 \left(\frac{i_n}{b/2}\right)^2 \frac{\rho}{2}\, v\, \omega_y \qquad (3b)$$

A roll is further opposed by a damping moment in roll which again is almost exclusively due to the wing. The proportion of the horizontal and vertical tail surfaces is altogether subordinate, besides being difficult to analyze mathematically because of its probable effect by the downwash of the rotating wing.

The "damping in roll" due to the wing is

$$K_{\omega_x} = \frac{c'_{n_\infty}}{1 + \frac{2c'_{a_\infty}}{\pi\,\Lambda}}\, F\, \frac{b^2}{12}\, \frac{\rho}{2}\, v\, \omega_x \qquad (4a)$$

The counterpart of these three aerodynamic moments K_T, K_{ω_y}, and K_{ω_x} is the acceleration in roll in unsteady attitude, and with whose mass effect they must balance. Accordingly, the equilibrium equation of the rolling moments is

$$\underline{\Sigma K = \frac{G}{g}\, i_y^2\, \dot{\omega}_x + K_{\omega_x} - K_T - K_{\omega_y} = 0;} \qquad (11)$$

the aerodynamic moments are inserted according to (2), (3b), and (4a).

<u>Moments of the vertical tail surfaces with free rudder.</u>- Thus far the equilibrium equations for the side loads and the yawing moments gave the proportion of the vertical tail surfaces in terms of constant rudder setting, that is, fixed lateral control. Now, as soon as this control is released, the rudder tries to balance the free moments acting on it. This case has never been treated in detail in previous publications.

In sideslipping an aerodynamic moment acts about the rudder axis, varying in magnitude according to the degree of balance, and which upon release of the lateral control displaces the rudder. If this is accompanied by a bank μ and a rate of turn $\dot{\psi}$, it produces a mass moment of the rudder about its hinge. Its amount varies with the magnitude of the resultant acceleration and its direction relative to the rudder axis (equations (5) and (7)). These two moments must mutually balance

$$c_{r_s} F_{s_r} t_{s_r} \frac{\rho}{2} v^2 = - M_{s_r} \left(\mu - \frac{v \dot{\psi}}{g}\right) \cos \varphi_0 \qquad (12)$$

where M_{s_r} is the weight moment exerted by the rudder about its axis if placed horizontally.

The air-flow direction at the vertical tail surfaces changes comparatively slowly during a lateral motion of disturbance, hence the rudder can adjust itself practically without inertia. The friction in the whole assembly can be kept to a minimum.

At the left side of (12) the coefficient c_{r_s} is dependent on the angle of air flow and the rudder displacement. According to wind-tunnel tests both can be expressed in linear relations

$$c_{r_s} = c'_{r_s} (\alpha_s) \alpha_s + c'_{r_s} (\beta_s) \beta_s$$

At the right side of (12) the angle $\left(\mu - \frac{v \dot{\psi}}{g}\right)$ of acceleration resultant and rudder axis is obtained from equation (8). The side load Z_T (according to (6)) is, with released rudder, also dependent on the rudder setting. But this changeable tail proportion is quite insignificant compared to the other components, so that its inclusion does not seem warranted. Thus we may put

$$- \left(\mu - \frac{v \dot{\psi}}{g}\right) \cos \varphi_0 = \frac{\frac{\rho}{2} v^2}{G/F} c'_q (\tau) \tau$$

at the right side of (12), where $c'_q (\tau)$ is the increment of the side load coefficient with the angle of sideslip for the whole airplane.

The total yawing moment of the vertical tail surfaces is

$$L_s = c_{n_s} F_s l_s \frac{\rho}{2} v^2 \quad (14)$$

where c_{n_s} is again dependent on the angle of air flow and the control movement. Both can be expressed in linear relation:

$$c_{n_s} = c'_{n_s}(\alpha_s)\,\alpha_s + c'_{n_s}(\beta_s)\,\beta_s$$

When combining (12) and (14) the angle β_s cancels out. The angle of air flow α_s is the sum of the angle of sideslip τ and angle of damping $l_s\omega_y/v$. With the abbreviations

$$m_{s_r} = \frac{M_{s_r}}{F_{s_r}\, t_{s_r}\, \frac{G}{F}}$$

$$\kappa = 1 - \frac{c'_{n_s}(\beta_s)}{c'_{n_s}(\alpha_s)}\,\frac{c'_{r_s}(\alpha_s)}{c'_{r_s}(\beta_s)}$$

the moments of the directional stability and of the damping in yaw with released rudder in the yawing moment equation (10) read as

$$L_\tau^\epsilon = \left[\frac{\partial c m_{s_0}}{\partial \tau} F\, b + \frac{\partial c_{n_s}}{\partial \beta_s}\,\frac{m_{s_r}\, a'_q(\tau)}{c'_{r_s}(\beta_s)} F_s\, l_s + \right.$$

$$\left. + \frac{\partial c_{n_s}}{\partial \alpha_s}\,\kappa\, F_s\, l_s\right] \frac{\rho}{2} v^2\, \tau \quad (1b^\epsilon)$$

$$L_{\omega_y}^\epsilon = \left[c d_{s_0} F\, b^2 + \frac{\partial c_{n_s}}{\partial \alpha_s}\,\kappa\, F_s\, l_s^2\right] \frac{\rho}{2} v\, \omega_y \quad (9b^\epsilon)$$

Thus releasing the rudder generally lowers the damping in yaw ($9b^\epsilon$) according to the degree of aerodynamic characteristics of the rudder (factor $\kappa < 1$). The directional stability ($1b^\epsilon$) is decreased in the same measure on one hand, while on the other, the mass moments of the rudder are increased. This is in sensible agreement with

Blenk's (reference 23) results for longitudinal stability with elevator released. Admittedly, the contributory effect of the mass moment at the elevator depends on the dynamic pressure, that is, on wing loading and flight attitude, but for the rudder only on the wing loading, since there is no direct interdependence between rudder displacement and dynamic pressure. The stabilizing effect of the mass moment may become significant when the rudder is well balanced aerodynamically (positive m_{s_r} and small $c'_{r_s}(\beta_s)$ and the side areas of the airplane are large ($c'_q(\tau)$ large).

As ($1b^{\epsilon}$) and ($9b^{\epsilon}$) contain nothing new compared with (1b) and (9b) for fixed rudder, all reference thereto is left to the section discussing the stability equations. The derivation of the stability equations is the same as for the fixed rudder.

<u>The general stability equations of disturbance in yaw.</u>- The three equilibrium equations of yaw (8), of yawing moments (10), and of rolling moments (11) together form a system of equations which gives the stability equations. The method of resolution is merely indicated so far as definitely necessary for a comprehensive understanding.

The rate of yaw ω_y and the rate of roll ω_x in the two moment equations follows from the superposition of two different rotations each. Their first summand is the time rate of change of the position of the plane of symmetry of the airplane to the flight path, that is, the rate of change of sideslip ($\dot{\tau}$) for the axis of yaw, the rate of change of angle of bank ($\dot{\mu}$) for the axis of roll. With their second summand, they are by way of path inclination φ_0 dependent on the angular velocity $\dot{\psi}$ of the c.g. of the airplane about the vertical axis, that is on the ensuing turn of the airplane. This is readily illustrated by visualizing the two limit cases: With level flight path the airplane yaws in a flat turn; with steep downward path the rotation of the airplane in the spiral motion is almost exclusively a roll. These relations are expressed in

$$\left.\begin{aligned} \omega_y &= \dot{\tau} + \dot{\psi}\cos\varphi_0 \\ \omega_x &= \dot{\mu} - \dot{\psi}\sin\varphi_0 \end{aligned}\right\} \qquad (16)$$

For this elementary case the rate of ω_y and ω_x can therefore be replaced by the derivatives of the variables τ and

μ already existent in the differential equations and by components of ψ. These three quantities are the real direct variables of the yaw.

Having written (18) in the equilibrium equations, the formal derivation of the stability equations offers nothing of particular interest.* But by virtue of the retained lateral air load and path slope the result manifests a somewhat greater completeness than afforded by Fuchs and Hopf's reductions (reference 6) without losing on clearness.

Table I is a survey of "time factors" obtained after reduction of the dynamic equilibrium equations to pure time equations.

The coefficients of the "principal equation" derived from the solution formula are:

$$\left.\begin{aligned}
B &= \underline{\underline{(k_{\omega_x}} + l_{\omega_y})} + z_T \\
C &= \underline{\underline{(k_{\omega_x}\, l_{\omega_y}} - k_{\omega_y}\, l_{\omega_x}) + l_T} + z_T\,(k_{\omega_x} + l_{\omega_y}) \\
D &= \underline{\underline{(k_{\omega_x}\, l_T} - k_T\, l_{\omega_x}) + \underline{\frac{g}{v}}\,(\underline{k_T}} \cos \varphi_0 - l_T \sin \varphi_0) + \\
&\qquad + z_T\,(k_{\omega_x}\, l_{\omega_y} - k_{\omega_y}\, l_{\omega_x}) \\
E &= \underline{\underline{\frac{g}{v}\,[(k_T\, l_{\omega_y} - k_{\omega_y}\, l_T)}} \cos \varphi_0 - \\
&\qquad - (k_{\omega_x}\, l_T - k_T\, l_{\omega_x}) \sin \varphi_0]
\end{aligned}\right\} \quad (21)$$

(Principal quotas and decisive summands are underscored.)

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*The second may be safely eliminated as time unit without untoward effect on the general validity as done by Fuchs and Hopf (reference 6) and others.

TABLE I

"Time Factors" of the Dynamic Equilibrium Equations

Notation	Signifies	Units	Time factor for	Derived from
s_T	$\frac{g}{v}\frac{q}{G/F}\left[-c''_{q_0}\frac{F_0}{F}+c'_{n_s}\frac{F_s}{F}+c_w\right]$	s^{-1}	Lateral air load	(6)
l_T	$\frac{g}{b}\left(\frac{b}{i_y}\right)^2\frac{q}{G/F}\left[c'_{m_{s_0}}+c'_{n_s}\frac{F_s}{F}\frac{l_s}{b}\right]$	s^{-2}	Stability in yaw	(1)
l_{ω_y}	$\frac{g}{v}\left(\frac{b}{i_y}\right)^2\frac{q}{G/F}\left[c_{d_{s_0}}+c'_{n_s}\frac{F_s}{F}\left(\frac{l_s}{b}\right)^2\right]$	s^{-1}	Damping in yaw	(9)
l_{ω_x}	$\frac{g}{v}\left(\frac{b}{i_y}\right)^2\frac{q}{G/F}\left[\frac{c'_{(t)}}{12}\right]$	s^{-1}	Yawing moment in roll	(4b)
k_T	$\left.\begin{matrix}\frac{g}{b}\left(\frac{b}{i_x}\right)^2\frac{q}{G/F}\times\\ \left[c'_{n_{q_0}}+\frac{1}{2}\frac{c'_{a_\infty}}{1+\frac{2c'_{a_\infty}}{\pi\Lambda}}\frac{s_n}{b/2}\nu\right]\end{matrix}\right\}$	s^{-2}	Stability in roll	(2)
k_{ω_y}	$\frac{g}{v}\left(\frac{b}{i_x}\right)^2\frac{q}{G/F}\left[\frac{c_n}{2}\left(\frac{i_n}{b/2}\right)^2\right]$	s^{-1}	Rolling moment in yaw	(3)
k_{ω_x}	$\frac{g}{v}\left(\frac{b}{i_x}\right)^2\frac{q}{G/F}\left[\frac{1}{12}\frac{c'_{a_\infty}}{1+\frac{2c'_{a_\infty}}{\pi\Lambda}}\right]$	s^{-1}	Damping in roll	(4a)

To insure dynamic stability for the whole yawing motion, the four coefficients B, C, D, and E and Routh's discriminant $R = BCD - D^2 - B^2 E$ must be positive. This condition is readily met with B and C where the damping in roll (k_{ω_x}) and yaw (l_{ω_y}), always positive in level flight, are decisive, and equally with D where a positive value for directional stability (l_T) is a primary requisite. The condition $R > 0$ for D is maintained within two limits, namely, $D \lessgtr BC (1 - E/C^2)$ and $D \gtrless BE/C$. The first boundary zone lies far beyond the practical range of normal flight, while the second, not mentioned at all by Fuchs and Hopf, is of considerable aeromechanical significance. With fulfillment of the stability conditions $B > 0$, $C > 0$, $E > 0$ the limit $R = 0$ already falls short before $D = 0$ is reached. By virtue of this relationship, the conditions of the dynamic lateral stability for all normal flight attitudes can be combined in the double inequation

$$\frac{C}{B} D > E > 0 \tag{22}$$

wherein B and C may always be presumed as being positive.

The stability condition $E > 0$ is most difficult to meet; the whole discussion of lateral stability is governed by it. This condition, first described by Reissner (reference 2) on the basis of purely static equilibrium consideration and subsequently elaborated on by Gehlen (reference 4), is unaffected by inertia effects and assumes the same significance in yaw as the static stability in roll in the theory of disturbance in roll. For this reason it is called "static lateral stability."

III. STATIC LATERAL STABILITY

Among the five generalized stability equations of disturbance in yaw, the "static lateral stability" $(E > 0)$) assumes a particular significance. This condition is expressed with two principal summands of equal value but different signs. Consequently, its numerical value cannot manifest very high absolute values and is, moreover, very sensitive to small changes in one of its components, so that a detailed analysis appears justified as well as necessary.

Flight path with static lateral instability.- In order to afford a survey of the procedure for obtaining static lateral stability independent of the mathematical considerations, the course of disturbance forced under the effect of lateral instability is briefly described.

Assume that an airplane flying level experiences from any cause of disturbance a change in bank, say, in the sense of a left bank. As the lift resultant continues in the plane of symmetry of the airplane, a lateral weight component is set up which effects a left sideslip. Because of its directional stability the airplane now attempts to turn left into the lateral sideslipping wind, while the rate of yaw is bounded by the opposing damping in yaw. At the same time there is a rolling moment in yaw, proportionate to the rate of yaw, which tends to turn the airplane farther into the produced bank, whereas the stability in roll promotes an opposing, that is, right-hand rolling moment out of the sideslip. With directional instability the accelerating moments exceed the decelerating ones. The turn into the sideslip receives not enough damping to allow the rolling moment due to stability in roll to successfully oppose the growth of the rolling moment in yaw. The bank and sideslip continue and the initial motion of disturbance develops into a turn with increasing bank and path curvature if the pilot, say, when flying without turn indicator, does not take timely control action.

The extent to which this so-called "spiral dive" may actually develop, is impossible to estimate with the method for small disturbances applied here. The rolling moment in yaw becomes markedly dependent on the bank; the balance of the path-normal forces is modified as a result of the rise in path curvature, where the now equally disturbed rolling moments appear, which also can effect the path curvature appreciably. The analysis of combined yaw and roll and its equilibrium aspects is extremely complex. The first successful attempt in Germany was made by v. Baranoff and Hopf (reference 9), and still promises to reveal much information about the origin and behavior of dangerous attitudes in "blind flying."

But, after all, the unintentional development and the initial course of spiral motion is contingent upon the condition of static lateral stability. In that respect, the often employed expression of a negative value of the static

lateral stability as "spiral instability" is legitimate.

Effect of flight attitude and structural quantities on the stability equation.- The result of retaining the initial path inclination in the simplified equilibrium equations is a fact which Fuchs and Hopf merely touched upon, although it is expressly noted in English reports (references 12 and 16). Static lateral stability ($E > 0$) is more difficult to obtain in climbing ($\varphi_0 > 0$) than in normal glide or level power flight at the same speed ($\varphi_0 \lessgtr 0$, c_n below the minimum gliding angle). The practical significance of this fact is hard to estimate in its extent so long as no flight data or mathematical data about the total effects, inclusive of climbing, for example, slipstream effect, are available in systematically collected and evaluated form. The existence of static lateral stability with level flight path ($\cos \varphi_0 = 1$, $\sin \varphi_0 = 0$) at normal lift coefficients is in any case, indication of increased static lateral stability for gliding at the same speed. In consequence of this the closer investigation may be restricted to level flight.

The insertion of the physical quantities into inequation (25), which gives the data of table I, affords a direct survey. All factors which are consistently positive may be omitted. The advantages of freedom of measure being here inferior to those of a clear formula, we analyze the equation of static lateral stability for level flight in the form of

$$\left[\left(c'_{m_{q_0}} + \frac{c'_{a_\infty}\, f_M}{2}\, \frac{s_u}{b/2}\, \nu\right) F\, b\right] \left[c_{d_{s_0}}\, F\, b^2 + c'_{n_s}\, F_s\, l_s{}^2\right] -$$

$$-\left[\frac{c_n}{2}\left(\frac{f_n}{b/2}\right)^2 F\, b^2\right]\left[c'_{m_{s_0}}\, F\, b + c'_{n_s}\, F_s\, l_s\right] > 0 \qquad (23a)$$

(Where abbreviated, $f_M = \dfrac{1}{1 + \dfrac{2 c_{a_\infty}}{\pi \Lambda}}$ (18)).

The first summand with positive sign, that is, stabilizing effect, is the product of static stability in roll (equation 2) and damping in yaw (equation (9b); the second, negative, that is, destabilizing summand, contains the rolling effect in yaw (equation (3b) and the static directional stability (equation 1b).

Efficient static stability in roll can only be obtained with appropriate wing design. The dihedral (ν) is a decisive factor. A similar, although inferior effect is afforded by the sweepback (contribution in $c'_{m_{q_0}}$). Wind-tunnel test data on this subject are scarce and confined to plan forms of constant chord (reference 17). Systematic investigations of other forms (as of trapezoidal wings, for instance) are not known. According to the measurements heretofore it may be inferred that, with small dihedral and sweepback, the rolling moments increase linearly with the sideslip so that sweepback and dihedral can, to a certain extent be mutually changed.*

The damping in yaw comprises the quota of wing, fuselage, and other parts, which is less affected by structural measures and therefore may be assumed as being practically invariable for a stated airplane form and the largely predominating proportion of the vertical tail surfaces which increases linearly with the vertical tail area and as the square of the length of tail. The ratio of total damping in yaw to the cited variable quota of the vertical tail surfaces.

$$\delta = \frac{c_{d_{s_0}} F\ b^2 + c'_{n_s} F_s\ l_s^2}{c'_{n_s} F_s\ l_s^2} = 1 + \frac{c_{d_{s_0}} F\ b^2}{c'_{n_s} F_s\ l_s^2} \qquad (24)$$

may be called the factor of the "relative damping in yaw." In order of magnitude, it should be estimated at between 1.2 and 1.4.

The effect of the rolling moment in yaw is, because of factor c_n, dependent on the flight attitude. With increasing lift coefficient, that is, decreasing dynamic pressure, its rise is approximately linear. There is a small balance in the variable normal force distribution, whose density in normal flight attitudes with increasing lift coefficient usually rises faster at the inside part of the wing than at the tips, and as a result of which the factor $\left(\frac{i_n}{b/2}\right)^2$ is gradually decreased.

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*The increase of rolling moments of a backswept wing in yaw should be markedly dependent on the profile camber, although there are no experiments known to confirm it. With the conventional wing section, a 1° dihedral corresponds to about 2° to 3° sweepback.

The static directional stability consists of the fixed quota of the fuselage and that of the vertical tail surfaces, which readily changes with the area of these control surfaces. The ratio of total directional stability to the variable vertical tail-surface quota

$$\sigma = \frac{c'_{m_{s_0}} F b + c'_{n_s} F_s l_s}{c'_{n_s} F_s l_s} = 1 + \frac{c'_{m_{s_0}} F b}{c'_{n_s} F_s l_s} \tag{25}$$

is called the factor of the "relative directional stability." With directional instability due to inherently unstable body and insufficient vertical tail surfaces $\sigma < 0$; with directional stability but unstable body proportion $0 < \sigma < 1$; with directional stability and inherently stable fuselage quota $\sigma > 1$. Considerable practical significance attaches to the "relative directional stability" insofar as the "static rudder effect," that is, the change of a steady sideslip with the rudder deflection $(\partial \tau / \partial \beta_s)$, is directly proportional to the reciprocal value of the relative directional stability.

Design measures for improving the static lateral stability.- The purpose of static lateral stability is to prevent a minor disturbance from developing in a bank with increasing or even constant, increased path curvature, and to foster a return to the steady, initial attitude with decreasing path curvature. In other words, the curvature reducing moments must be augmented and the curvature increasing moments moderated. Consequently, the means of obtaining static lateral stability are: good stability in roll and high damping in yaw, together with very low rolling moment in yaw and appropriate static directional stability.

The requisite structural measures become readily apparent from the inequation (23a). The first bracketed factors on both sides express the rolling moment effects which are almost exclusively governed by the wing design. The two second bracketed factors contain the quota of the yawing moments, which are almost exclusively contingent upon the design of the fuselage and the vertical tail surfaces. The primary requisite in wing design is flight performance and longitudinal stability. The extent to which the airplane designer may use the wing section for obtaining directional stability, depends upon the inaccessi-

bility of other measures. Hence we shall first analyze the furtherance of directional stability through influencing the yawing moments and then ascertain how static lateral stability may be obtained by acting upon the rolling moments.

a) Action upon yawing moments.- On the stabilizing side of inequation (23a) we have the damping in yaw, and on the destabilizing side, the stability in yaw. With (24) and (25), inequation (23a) can be written as

$$\left[c'_{m_{q_0}} + \frac{c'_{a_\infty} f_M}{2} \frac{s_u}{b/2} \nu\right] \left[\delta \frac{l_s}{b}\right] > \left[\frac{c_n}{2} \left(\frac{i_n}{b/2}\right)^2\right] \left[\sigma\right] \quad (23b)$$

One noteworthy fact here is that it is not the absolute amount of the directional stability that matters but rather its relative value with respect to the stability quota of the vertical tail surfaces, so that good directional stability is not always eo ipso incompatible with the requirement for static lateral stability.

Now we analyze the appropriate measures on fuselage and vertical tail surfaces for an airplane whose principal dimensions are assumed as given in design, model, or even in first construction, but which lacks altogether or is deficient in lateral stability.

For structurally affecting the yawing moments, there are the rise in cross-wind force and the vertical tail area (c'_{n_s}, F_s) and the length of the tail (l_s). The aerodynamic efficiency of the vertical tail surfaces, proportionate to the product $c'_{n_s} F_s$, may sensibly be considered only as function of its quantity unless its own plan form is also radically modified.

The magnitude of the vertical tail surfaces is given in (23a) on the positive side in damping in yaw and on the negative side in the directional stability. Both effects change linearly with the control area; but the directional stability contains further the fuselage quota, the damping in yaw the wing quota as constant summands. Enlarging the vertical tail surfaces while retaining the same length of tail, increases the lateral stability when

$$\frac{\partial}{\partial F_s}\left(\frac{c_{d_{s_0}}\ F\ b^2 + c'_{n_s}\ F_s\ l_s^2}{c'_{m_{s_0}}\ F\ b + c'_{n_s}\ F_s\ l_s}\right) > 0 \quad \text{(from inequation 23c).}$$

This condition is met when (with (24) and (25))

$$\sigma \gtrless \delta .$$

Aside from the comparatively rare case of high inherently stable fuselage ($\sigma > 1.2 \ldots 1.4$), enlargement of the vertical tail surfaces generally results in lower static lateral stability. This interdependence is explained with the varying relative quota of the vertical tail surfaces on damping in yaw and on directional stability. However, if only the fuselage is statically neutral or an unstable fuselage has correspondingly small fin and rudder, a noticeable damping in yaw exists nevertheless. Owing to this "lead" of the damping over the stability (noticeable even for inherently stable bodies up to the limit $\sigma = \delta$) any enlargement of the vertical tail surfaces affects the directional stability relatively more than damping in yaw, and the result is less lateral stability.

The absolute directional stability may, without harming the lateral stability equation, become so much higher as the "lead" of the damping over the stabilizing effect of the vertical tail surface change is greater. From this point of view, it is propitious to consider the body quota as unstable, then the vertical tail area needed up to neutral stability supplies (because of its great distance from the center of gravity) a more effective degree of damping than the side area of an inherently stable fuselage. The factor of the relative directional stability which governs the lateral stability, is here always less than 1, because even the highest absolute directional stability about the unstable fuselage quota is less than that of the vertical tail surfaces (equation 25). In that case insufficient lateral stability may occasionally, without raising the stability in roll, be improved by sacrificing a considerable portion of directional stability through reduction in fin and rudder area.

Length of tail enters according to inequation (23a) the static directional stability linearly, and the damping in yaw, squared. Whichever effect predominates for lateral stability depends again on the constant quotas of wings and fuselage. Lengthening the tail without modifying the vertical tail surfaces gives a higher lateral sta-

bility when $\frac{\partial}{\partial l_s}\left(\frac{c_{d_{s_0}} F b^2 + c'_{n_s} F_s l_s^2}{c'_{m_{s_0}} F b + c'_{n_s} F_s l_s}\right) > 0.$

This condition is met when (with (24) and (25))

$$\sigma > \frac{\delta}{2}.$$

Consequently, lengthening the tail is not always followed by higher lateral stability. With unstable quota of fuselage toward directional stability $(\sigma < \frac{\delta}{2} < 1)$ the lateral stability may appear poorer, which - without modification of stability in roll - can only be corrected by simultaneously reducing the area of fin and rudder. Lengthening the tail while maintaining directional stability (σ) by reducing the vertical tail area accordingly, is always followed by improved lateral stability according to inequation (23b).

Lengthening the tail for the purpose of increasing lateral as well as directional stability without modifying the fin and rudder is successful only when the directional stability of the airplane in the initial condition is in excess of approximately two thirds of the stabilizing tail control quota alone $(\delta/2)$. Otherwise the stability in roll must be raised at the same time.

The effectiveness of the structural measures for favorable effect on the yawing moments was analyzed on the premises of constant stability quota of fuselage with respect to the center of gravity of the airplane. On the other hand, any enlargement of the vertical tail surfaces or lengthening of the tail tends to shift the c.g. toward the rear. The center of pressure of the lateral air loads without vertical tail surfaces lies usually closely ahead or behind the c.g., hence its stability quota is very sensitive to changes in weight distributions; compared to it the insignificant rearward displacement of the center of pressure by lengthening the rear end of the fuselage is of secondary importance. In any case, it should be remembered that structural measures for influencing the yawing moments will not bring the anticipated results unless the ensuing weight displacements are carefully analyzed.

<u>b) Influencing the rolling moments</u>.- The two rolling moments in (23a) contain the aspect ratio of the wing as

the only common design quantity. The moment of the stability in roll grows linearly with the wing span, that is, as the square of the aspect ratio. Contrariwise, the rolling moment in yaw increases as the square of the span, that is, linearly with the aspect ratio. Consequently, the enlargement of the aspect ratio vitiates the static lateral stability. To be sure, the concurrent rise of f_M and δ on the stabilizing side (23b) moderates this disadvantage to a certain degree.

Other effective relations of structural nature do not exist between the two rolling moments. Lateral stability is decidedly affected by the dihedral, whereas the rolling moment in yaw is governed by the momentarily existent normal-force coefficient, that is, the attitude of flight.

The static lateral stability is linearly decreased with increasing lift coefficient as a result of the rolling moment in yaw, consequently behaves utterly unlike the approximately constant and rather increasing longitudinal stability at higher lift coefficients. This fact imposes a certain limitation of the requirement for static lateral stability so long as no means are found for counteracting the rise of rolling moment in yaw with the lift coefficient. Until such time, a neutral stability limit at around the lift coefficient of best gliding ratio and a minor instability at higher lift coefficients will have to be considered as satisfactory.

The drop of static lateral stability with increasing lift coefficient can be closely restricted by influencing the spanwise lift distribution. The most conventional method is to twist the wing (reduction of $(i_n/b/2)$. But the disadvantage of this method usually is its vitiating effect on the induced wing drag, so that its effectiveness is at the expense of poorer performance in flight. For tapered wings without geometrical twist the rolling moments in yaw are already lower than for the rectangular wing, although twisting would not be very effective. Consequently, tapered wings without twist are accordingly about equivalent to rectangular wings with moderate twist.

The principal condition of lateral stability, compared to which all attempts at influencing the yawing moments and the rolling moments in yaw are no more than auxiliary measures, is ample lateral (roll) stability. Only through it together with damping in yaw - always positive in normal

flight - can the stabilizing left side of inequation (23a) retain its necessary predominance. There is no other practical way, as the directional stability on the right-hand side should also always have positive sign.

That the dihedral is the most effective way of obtaining any static lateral (roll) stability, has been known since the beginning of aviation. The amount of dihedral necessary for static roll stability is

$$\nu \geq \frac{c_n}{c'_{a_\infty}\ f_M} \frac{\left(\frac{i_n}{b/2}\right)^2}{\frac{s_u}{b/2}} \frac{b}{l_s} \frac{\sigma}{\delta} - \frac{2c'_{m_{q_0}}}{c'_{a_\infty}\ f_M\ \frac{s_u}{b/2}} \qquad (26a)$$

according to inequation (23b).

One method frequently resorted to, is to fit a straight center section, usually rectangular, to the dihedrally arranged wing tips. The effectiveness of this method is similar to the static effectiveness of ailerons of less than semispan length (reference 22). The center section span b_i is very seldom in excess of one fourth of the total span b; within this limit the stability quota of the dihedral varies in close approximation as the factor $\left[1 - \left(\frac{b_i}{b}\right)^2\right]$, which is amply sufficient for design calculations.

The value of ratio $\left(\frac{i_n}{b/2}\right)^2 : \frac{s_u}{b/2}$ varies with the plan form of the wing, the corresponding lift distribution, and twist. The lowest possible practical limit is perhaps reached with a rectangular, twisted wing and elliptical lift distribution where the form ratio approaches one half. The upper limit can be reached by a tapered wing with noticeable lift distribution disturbance in the center at about three fourths form ratio. For cursory calculations the value two thirds valid for a rectangular wing with rectangular lift distribution, can be considered as a satisfactory average. It even affords ample security when the lift distribution in the center is held to a minimum of disturbance. Putting in that case $c'_{a_\infty} f_M \approx c'_a$ and $\frac{s_u}{b/2} = \frac{1}{2}$ gives the approximation formula

$$\nu \geq \frac{\frac{2}{3} c_n \frac{b}{l_s} \frac{\sigma}{\delta} - 4 c'_{m_{q_0}}}{c'_a \left[1 - \left(\frac{b_1}{b}\right)^2\right]} \quad \text{(in radians)} \tag{26b}$$

For the special case of $\frac{\sigma}{\delta} = 1$, $c'_{m_{q_0}} = 0$ and $\frac{b_1}{b} = 0$, it agrees with Reissner's formula of 1910 (reference 2).

The lateral stability quota to the whole arrangement of the wing without dihedral $(c'_{m_{q_0}})$ is probably positive in most cases. It comprises a number of mathematically intractable single effects in addition to that of the sweepback. According to German and foreign wind-tunnel tests (reference 17) even wings without dihedral and sweepback experience minor rolling moments in yaw, especially at higher angles of attack. Besides, the rolling moments of the wing are affected by the fuselage and in sideslipping, the vertical tail surfaces likewise generate rolling moments. In special designs (as of seaplanes, for instance), floats under the center of gravity, engine nacelles above the center of gravity, etc., may also contribute substantially.

In this connection, the effect of sweepback merits particular attention. Although being narrowly confined, because of the longitudinal stability requirements, the sweepback is always desired for lateral stability. A change in sweepback is often followed by a shift of the center of gravity in the same direction, so that the directional stability changes also. Therefore the total effect on the lateral stability can - as when influencing the yawing moments - only be estimated with due allowance for eventual weight displacements.

<u>Static lateral stability with released rudder</u>.- The release of the rudder effects a change in static directional stability and damping in yaw. (See section II, page 16, last paragraph.) Hence $(1b^\epsilon)$ and $(9b^\epsilon)$ can be inserted in (23a) in place of (1b) and (9b). The static lateral stability is increased when

$$\frac{c_{d_{s_0}} F b^2 + c'_{n_s} \kappa F_s l_s^2}{c_{d_{s_0}} F b^2 + c'_{n_s} F_s l_s^2} > \frac{c'_{m_{s_0}} F b + c'_{n_s} (\kappa + m_{s_r} \xi) F_s l_s}{c'_{m_{s_0}} F b + c'_{n_s} F_s l_s}$$

where (for reasons of clearness)

$$\xi = \frac{c'_q(\tau)}{c'_{r_s}(\beta_s)} \frac{c'_{n_s}(\beta_s)}{c'_{n_s}(\alpha_s)}.$$

It is readily ascertained in two specific cases whether the release of the rudder increases or decreases the static lateral stability. When the mass effect of the rudder is negligibly small the release of the rudder normally acts as a reduction in vertical tail area $(\kappa < 1)$ from F_s to $\kappa'' F_s$; consequently, the static lateral stability becomes usually $(\sigma < \delta)$ (section III, page 27) greater than with locked rudder. When the fuselage quota to the static directional stability is neutrally stable, the mass effect of the rudder may become more evident. In that case, the above generalized equations should ascertain any increase in static lateral stability when (in conjunction with (24) and the κ values derived from (14)

$$m_{s_r} < \frac{\delta - 1}{\delta} \frac{c'_{r_s}(\alpha_s)}{c'_q(\tau)}$$

Thus it is seen that the mass effect of the rudder may occasionally have also a destabilizing effect, especially when the rudder is aerodynamically well balanced $(c'_{r_s}(\alpha_s)$ small).

The difference in lateral stability with released and locked rudder can therefore be influenced by appropriate proportioning of air load and mass balance. The air-load balances can be obtained in the usual manner on the rudder direct or with an auxiliary surface suitably connected kinematically with the principal rudder. The balance of the mass moments always promotes lateral stability. Even an "overbalance" for reversing the mass moments may be advantageous if not involving other difficulties. Whereas no great accuracy attends any preliminary computation of the effects, this analysis should at any rate prove of use in the application and effectiveness of structural measures toward improving the lateral stability of an airplane with released rudder.

<u>Static lateral stability in power flight</u>.- For the present the slipstream effects on the rolling and yawing

moments can only be estimated in direction and order of magnitude because of the lack of reliable measurements.

Compared to gliding flight, the type of equilibrium condition changes in power flight. But the asymmetries due to engine torque and slipstream turbulence can be disregarded at first in the stability analysis.

The increased mean flow velocity in the slipstream relative to the flight speed is with the conventional engine installations principally effective on the vertical tail surfaces and thus affects the directional stability and the damping in yaw. With equilibrium disturbances the change in angle of air flow corresponds on the vertical tail surfaces to the ratio of rate of sideslip to forward speed with respect to the propeller slipstream. This ratio is smaller with power on than with power off. On the other hand, there is a higher mean dynamic pressure on the vertical tail surfaces, corresponding to the square of the relative speed increase in the propeller slipstream. Altogether, then, the effect on the lateral stability with power on should not be unlike that on vertical tail surfaces enlarged approximately in ratio of mean propeller slipstream velocity to flying speed. For equal forward speed the airplane should, in general, therefore, appear less statically stable laterally than in gliding.

The extent to which these conclusions correspond for the different structural designs and power-plant installations, must be decided in flight tests for each case.

IV. THE BOUNDARIES OF DYNAMIC LATERAL STABILITY

Although being the most important, static lateral stability is not the sole sufficing condition for dynamic lateral stability. The derivation of the stability conditions (section II, page 19) shows them to be five in number. Three ($B > 0$, $C > 0$, and $E > 0$) must be met independently from each other, while the other two ($D > 0$ and $R > 0$) mutually bound each other and may be combined into $\left(D > \frac{B\,E}{C}\right)$.

The first two conditions ($B > 0$ and $C > 0$) require principally positive damping in roll and in yaw. They are

fulfilled in normal flight and also at higher angles of attack as far as the validity range of "small oscillations" extends at all, so in this case their importance as stability boundaries is negligible. Nevertheless, it should be remembered that all stability conditions can be damaged simultaneously when the damping in roll becomes negative (intensified in stalling). A disturbance during such an unstable equilibrium condition is followed by incipient spinning. Complete loss of damping in yaw is hardly to be expected in airplanes of conventional design, while its extent as concerns tailless types, due to negative tangential loads at high angles of attack, must be left to future model and flight tests.

In most instances, the "static lateral stability" $(E > 0)$ suffices to insure also the dynamic stability. The first part of the double inequation (22) represents, however, a superior limit $\left(\frac{C}{B} D > E\right)$ which may become practically important under stated conditions. This boundary

$$\frac{C}{B} D \approx [k_{\omega_x} l_T + \left(\frac{g}{v} - l_{\omega_x}\right) k_T] \; l_{\omega_y} \tag{27}$$

roughly approximated from (21), consistently approaches lower values as the values of damping and stability in yaw and roll become less. Little damping and decreasing pitching stability are anticipated except at high angles of attack when the wing shows signs of separation of flow and negative tangential loads and the vertical tail surfaces appear blanketed; contrariwise, there is always insufficient directional stability when the fin and rudder are too small, irrespective of the flight attitude.

Dynamic stability $\left(\frac{C}{B} D > E\right)$ prevails so long as, according to (27) and (21), the condition

$$\frac{l_T}{k_T} > \frac{l_{\omega_x}}{k_{\omega_x} + \frac{g}{v} \frac{k_{\omega_y}}{l_{\omega_y}}} \tag{28}$$

is not infringed upon.

The yawing moment in roll (l_{ω_x}) being neither notice-

ably negative nor vanishingly small in normal flight, an expressed positive directional stability is the most effective assurance against dynamic lateral instability.

Further discussion of this stability condition is prefaced by a report on an American flying boat (reference 15), wherein the unstable oscillations occurring within this boundary zone of dynamic stability, are graphically described:

"The flying boat was designed for the specific purpose of experimenting with lateral controllability, the object being to develop the airplane which amateur pilots of little experience could use safely.

"For the first flights it was rigged with a 6° dihedral and with a vertical tail surface of 1.82 m² (19.6 sq.ft.). It became immediately apparent, however, that such a rigging produced the instability classified as 'unstable oscillations.' It was necessary to control the boat all the time by the rudder in order to fly on a straight line. When all the controls were held in neutral position at about cruising speed, the generation and mechanics of the oscillations became quite evident. The boat was observed to deviate from the straight course, banking more rapidly than turning, until it overbanked and sideslipped. Then it leveled out rather briskly and banked to another side, repeating the cycle. The flight consisted of a series of turns alternatively to the right and to the left, apparently quite steady and persistent, without any tendency either to increase or die out.* The banks reached about 30°. Average direction of the flight was maintained quite well, and while making these turns the airplane felt steadier and more secure than in normal flight with its odd tendency to hunting.

"For the next experiment the dihedral was reduced to 3°, the boat not being changed in any other way. It was found that it behaved much better but not really well yet. ... The oscillations were much more gentle, the bank on each turn not increasing beyond 20°. It was evident that

*According to this the term "unstable oscillation" commonly used in English-speaking countries is unintelligible, at least so far as this type of motion is concerned.

instability was much less pronounced, but still remained of the same type - 'unstable oscillations.'

"For a third experiment the vertical fin and rudder were changed, increasing the vertical tail area from 1.82 m^2 (19.6 sq.ft.) to 2.58 m^2 (27.8 sq.ft.). The dihedral remained the same and the boat itself was not changed in any way. The airplane was found to be very good on lateral controls and was flown for several hours in this condition. The use of ailerons and of the rudder was tried on very flat glides and they were found to be quite effective at low speed. This is particularly interesting because while the airplane had 'unstable' rigging, several pilots complained of very weak controls. It proves that it is unadvisable to pass judgment on controls until the lateral stability is properly adjusted."

The generation of the thus described "unstable oscillations" is also described by Gehlen (reference 4). The principal stability equation usually contains two real roots which mainly describe the aperiodic roll (damping in roll) and the aperiodic yaw (static lateral stability). The other two roots are mostly conjugated complex and define a damped oscillation in yaw contained within the yaw, caused by the static directional stability. Then, when the directional stability reaches a stated lower limit set by the damping in yaw, the yaw becomes aperiodic. Upon further decrease in directional stability the double root of the yawing motion resolves and a new conjugated complex pair of roots may form apart from the now real root of the aperiodic yawing motion. The pair of roots absorbs the hitherto aperiodic yawing moments and then appears as "unstable oscillation." The center of pressure oscillation may at first be dynamically stable. Not until further decrease in directional stability is the stability boundary $D \approx \frac{B}{C} E$ reached. The stable range of the unstable oscillation is very small. Since the damping in it is vanishingly small, its appearance in flight is already felt as start of dynamic instability.

The generation of unstable oscillations is practically impossible so long as the strongly damped yawing oscillation refrains from becoming aperiodic. The necessary condition for this is in first approximation $l_T > \frac{1}{4} l^2_{\omega_y}$ and gives as boundary a relative directional stability of the order of magnitude of $\delta \approx \frac{1}{30} \ldots \frac{1}{10}$; such low values,

possible only with unstable fuselage proportion, already practically indicate neutral directional stability. With locked rudder the directional stability is, moreover, mostly above that boundary range, although it may by releasing the rudder drop occasionally low enough to permit unstable oscillations to appear, so that the rudder must be held again. This occurrence is not unknown in practical flight.

The directional stability is - similar to the longitudinal stability - appreciably affected by the position of the center of gravity; it drops when the c.g. shifts to the rear. In most forward positions with maximum directional stability, the static lateral stability is lowest and no unstable oscillations are anticipated. With rearward position of the center of gravity, the static lateral stability increases, but then there is a possibility that vanishing directional stability may produce unstable oscillations.

However, for an airplane which is directionally stable with rudder released in all possible center-of-gravity positions, the static lateral stability is the only boundary of the dynamic lateral stability requiring particular attention.

V. THE USE OF WIND-TUNNEL TESTS FOR LATERAL STABILITY COMPUTATIONS

The wind-tunnel tests which the airplane designer may use for preliminary lateral stability analysis, fall into two groups: the determination of the force and moment coefficients on models suspended in the wind tunnel, and the determination of the rotation factors of models which are free to rotate in the wind tunnel. The conventional 6-component measurements fall in the first category. Their chief advantages are simplicity and inexpensiveness, but they afford only approximations as to the rotation effects on the free airplane. The second method is particularly developed in England. It entails considerable expense in suspension and test equipment and paper work, but it usually affords very satisfactory results (reference 20).

The airplane designer, especially in Germany, is as a rule only concerned with the first type of wind-tunnel tests, whether as basis or for checking a new design. The

mathematical and the empirical data available heretofore are quite scarce, largely unpublished, and not as easily transferable as in longitudinal stability investigations. As a result, it sometimes happens that new designs must be specially examined for each case without reliable comparable standards until clear empirical principles have been formulated for the design and the safe employment of available test data.

The correct interpretation of 6-component measurements to design and, if necessary, to design modifications, calls for a brief survey of the contributions of the most important structural parts to the forces and moments on the complete airplane. Because of the mutual interference between wing, body, and control surface the mathematical development from single summands is useful only in very rough approximation, and leaves many sources of error out of consideration. The resolution in difference measurements is far more expedient since, if correctly arranged, they not only include most sources of error in the effect but also permit detection at the source and thus offer an incentive to improved forms.

For the measurements defining the forces and moments in yaw the airplane model in its main form - size of wing, plan form, body shape, tail arrangement - may be considered fixed, once the design has advanced to the measuring stage. For defining the yawing and rolling moments, the size of fin and rudder and of the dihedral may, if necessary, be left open. Changeability of fin and rudder is readily insured on the model; changeability of dihedral requires a divided model wing and some means of adjusting which, however, is also quite easy.

A model of the hitherto conventional elementary design, that is, with detachable fin and rudder, and a one-piece, rigid wing, requires at least two sets of 6-component measurements for a survey: those on the complete model, and those without vertical tail surfaces. The first give, aside from the static rolling moments, the total absolute directional stability (c_{m_s}); the second give the quota of fuselage, wing, and other structural components to the static yawing moment ($c_{m_{s_o}}$). The difference of both is a criterion for the moment contribution of the vertical tail surfaces and its effect under the influence of the other structural parts. In this manner, it affords a more exact

definition of the relative directional stability (equation 25) which governs the lateral stability, than the mathematical determination from form, size, and position of the vertical tail surfaces.

All the data necessary for the production of any stability condition can be obtained on a model with variable vertical tail surfaces and variable dihedral. The roll stability without dihedral and its increase with the dihedral angle is one important factor therein. An airplane design, which after wind-tunnel tests is unsatisfactory in its original form, can thus be comparatively easily and safely modified.

The interpretation of the test data for the evaluation of the anticipated lateral stability, must first of all attest to the fulfillment of the two simple conditions

$$\left.\begin{aligned} c'_{m_q} &\gg 0 \\ c'_{m_s} &\gg 0 \end{aligned}\right\} \qquad (29)$$

for the extreme rear positions of the center of gravity provided in the design. In addition, the ratio of the stability in yaw and roll must meet the condition according to inequation (23a) or (23b). Transformed for the present purpose, it stipulates that

$$\frac{c'_{m_q}}{c'_{m_s}} > \frac{1}{2}\left(\frac{i_n}{b/2}\right)^2 \frac{c_n}{\delta\,(c'_{m_s} - c'_{m_{s_0}})\,\frac{l_s}{b}} \qquad (30a)$$

Here the increase of damping in yaw due to wing and fuselage $(\delta > 1)$ may be estimated or else disregarded altogether $(\delta \approx 1)$ as safety margin. The form factor $\left(\frac{i_n}{b/2}\right)^2$, representative of the spanwise normal force distribution, as criterion for the geometric moment of inertia of the normal force distribution figure (see section II, page 10), is also readily estimated after the wing has been designed. In the majority of cases it lies between 1/4 ... 1/3. The anticipated lateral stability can accordingly be estimated within the degrees of accuracy obtainable in wind-tunnel tests when putting $\frac{1}{2}\left(\frac{i_n}{b/2}\right)^2 \frac{1}{\delta} \approx \frac{1}{8}$. Then the condition assumes the following

form:

$$\frac{c'_{m_q}}{c'_{m_s}} \gtrapprox \frac{1}{8} \frac{c_n}{(c'_{m_s} - c'_{m_{s_o}}) \frac{l_s}{b}} \tag{30b}$$

A more accurate allowance for factors $\left(\frac{i_n}{b/2}\right)^2$ and δ is superfluous except in decidedly special cases.

If a model, after the first test results, does not meet condition (30a), the necessary enlargement of dihedral ($\Delta\nu$) is readily estimated according to equation (26a) or (26b) when substituting the measured quantity $\sigma = \frac{c'_{m_s}}{c'_{m_s} - c'_{m_{s_o}}}$ for the relative directional stability and the measured rolling moment increase c'_{m_q} for the roll stability ($c'_{m_{q_o}}$).

The form of the inequations (30) itself again attests the importance of the relative directional stability $\left(\sigma = \frac{c'_{m_s}}{c'_{m_s} - c'_{m_{s_o}}}\right)$ over the absolute directional stability as concerns the lateral stability. The more pronouncedly unstable the model without fin and rudder is ($c'_{m_{s_o}} < 0$), the more readily condition (30) may be met. A limit is set by the feasibility of designing vertical tail surfaces of a size large enough to assure ample absolute directional stability. In accordance with practical experience the value $\sigma \approx 1/3$ represents a minimum which should never be passed by commercial airplanes for reasons of safety.

The theoretical treatment of lateral stability is as old as aviation itself. The formulas of Ferber (1905) in France, Deimler (1910) in Germany, and Bothezat (1911) in France, contain omissions or erroneous assumptions about the aerodynamic moments due to the rate of rotations, thus leading to partly erroneous, partly incomplete results

(details in reference 4). The first comprehensive reports on lateral motion with clear results are those of Reissner (references 1 and 2), and of Bryan (reference 3), which became the basis of all subsequent developments. Then came Gehlen (1912) with his systematic treatise of the whole lateral stability problem; his doctor's thesis on Roll Stability and Lateral Control of Airplanes (reference 4) is a survey of greatest detail and technico-physical clearness, and which even today merits unrestricted recognition. The further development and the inclusion of these pioneer efforts in the compiled works of Bairstow (reference 5) in England, and Fuchs and Hopf (reference 6) in Germany, may be considered as the end of the first stage of development.

Subsequently Germany contributed various mathematical treatises (references 8, 9, and 10) on the generalization of the stability theory, while England received a much stronger impetus to further progress as a result of a multitude of wind-tunnel and flight tests. This accounts for the plentiful material published in the English periodicals in the last few years. At present, the safety requirements for "blind flying" appear to bring the discussions on lateral stability into the foreground again.

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